Chapter 16 – Random Variables

1. Expected value.

a) $\mu = E(Y) = 10(0.3) + 20(0.5) + 30(0.2) = 19$

b) $\mu = E(Y) = 2(0.3) + 4(0.4) + 6(0.2) + 8(0.1) = 4.2$

2. Expected value.

a) $\mu = E(Y) = 0(0.2) + 1(0.4) + 2(0.4) = 1.2$

b) $\mu = E(Y) = 100(0.1) + 200(0.2) + 300(0.5) + 400(0.2) = 280$

3. Pick a card, any card.

a)

Win	\$0	\$5	\$10	\$30
P(amount won)	$\frac{26}{52}$	$\frac{13}{52}$	$\frac{12}{52}$	$\frac{1}{52}$

b) $\mu = E(\text{amount won}) = \$0\left(\frac{26}{52}\right) + \$5\left(\frac{13}{52}\right) + \$10\left(\frac{12}{52}\right) + \$30\left(\frac{1}{52}\right) \approx \$4.13$

c) Answers may vary. In the long run, the expected payoff of this game is \$4.13 per play. Any amount less than \$4.13 would be a reasonable amount to pay in order to play. Your decision should depend on how long you intend to play. If you are only going to play a few times, you should risk less.

4. You bet!

a)

Win	\$100	\$50	\$0
P(amount won)	$\frac{1}{6}$	$\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{5}{36}$	$\left(\frac{5}{6}\right)\left(\frac{5}{6}\right) = \frac{25}{36}$

b) $\mu = E(\text{amount won}) = \$100\left(\frac{1}{6}\right) + \$50\left(\frac{5}{6}\right) + \$0\left(\frac{25}{26}\right) \approx \$23.61$

c) Answers may vary. In the long run, the expected payoff of this game is \$23.61 per play. Any amount less than \$23.61 would be a reasonable amount to pay in order to play. Your decision should depend on how long you intend to play. If you are only going to play a few times, you should risk less.

5. Kids.

a)

Kids	1	2	3
P(Kids)	0.5	0.25	0.25

b) $\mu = E(\text{Kids}) = 1(0.5) + 2(0.25) + 3(0.25) = 1.75$ kids

c)

Boys	0	1	2	3
P(boys)	0.5	0.25	0.125	0.125

$\mu = E(\text{Boys}) = 0(0.5) + 1(0.25) + 2(0.125) + 3(0.125) = 0.875$ boys

6. Carnival.

a)

Net winnings	\$95	\$90	\$85	\$80	-\$20
number of darts	1 dart	2 darts	3 darts	4 darts (win)	4 darts (lose)
P(Amount won)	$\left(\frac{1}{10}\right) = 0.1$	$\left(\frac{9}{10}\right)\left(\frac{1}{10}\right) = 0.09$	$\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right) = 0.081$	$\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right) = 0.0729$	$\left(\frac{9}{10}\right)^4 = 0.6561$

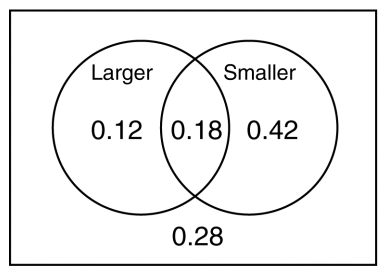
b) $\mu = E(\text{number of darts}) = 1(0.1) + 2(0.09) + 3(0.081) + 4(0.0729) + 4(0.6561) \approx 3.44$ darts

c) $\mu = E(\text{winnings}) = \$95(0.1) + \$90(0.09) + \$85(0.081) + \$80(0.0729) - \$20(0.6561) \approx \$17.20$

7. Software.

Since the contracts are awarded independently, the probability that the company will get both contracts is (0.3)(0.6) = 0.18. Organize the disjoint events in a Venn diagram.

Profit	larger only \$50,000	smaller only \$20,000	both \$70,000	neither \$0
P(profit)	0.12	0.42	0.18	0.28

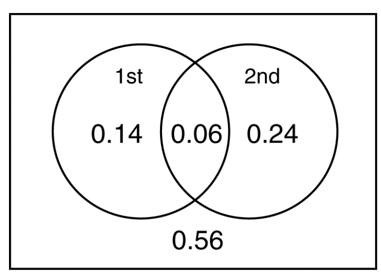


$$\mu = E(\text{profit}) = \$50{,}000(0.12) + \$20{,}000(0.42) + \$70{,}000(0.18) = \$27{,}000$$

8. Racehorse.

Assuming that the two races are independent events, the probability that the horse wins both races is (0.2)(0.3) = 0.06. Organize the disjoint events in a Venn diagram.

Profit	1st only \$30,000	2nd only \$30,000	both \$80,000	neither – \$10,000
P(profit)	0.14	0.24	0.06	0.56



$$\mu = E(\text{profit}) = \$30{,}000(0.14) + \$30{,}000(0.24) + \$80{,}000(0.06) - \$10{,}000(0.56) = \$10{,}600$$

9. Variation 1.

a)

$$\sigma^2 = Var(Y) = (10-19)^2(0.3) + (20-19)^2(0.5) + (30-19)^2(0.2) = 49$$

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{49} = 7$$

b)

$$\sigma^2 = Var(Y) = (2-4.2)^2(0.3) + (4-4.2)^2(0.4) + (6-4.2)^2(0.2) + (8-4.2)^2(0.1) = 3.56$$

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{3.56} \approx 1.89$$

10. Variation 2.

a)

$$\sigma^2 = Var(Y) = (0-1.2)^2(0.2) + (1-1.2)^2(0.4) + (2-1.2)^2(0.4) = 0.56$$

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{0.56} \approx 0.75$$

b)

$$\sigma^2 = Var(Y) = (100-280)^2(0.1) + (200-280)^2(0.2) + (300-280)^2(0.5) + (400-280)^2(0.2) = 7600$$

$$\sigma = SD(Y) = \sqrt{Var(Y)} = \sqrt{7600} \approx 87.18$$

11. Pick another card.

Answers may vary slightly (due to rounding of the mean)

$$\sigma^2 = Var(\text{Won}) = (0-4.13)^2\left(\frac{26}{52}\right) + (5-4.13)^2\left(\frac{13}{52}\right) + (10-4.13)^2\left(\frac{12}{52}\right) + (30-4.13)^2\left(\frac{1}{52}\right) \approx 29.5396$$

$$\sigma = SD(\text{Won}) = \sqrt{Var(\text{Won})} = \sqrt{29.5396} \approx \$5.44$$

12. The die.

Answers may vary slightly (due to rounding of the mean)

$$\sigma^2 = Var(\text{Won}) = (100-23.61)^2\left(\frac{1}{6}\right) + (50-23.61)^2\left(\frac{5}{36}\right) + (0-23.61)^2\left(\frac{25}{36}\right) \approx 1456.4043$$

$$\sigma = SD(\text{Won}) = \sqrt{Var(\text{Won})} \approx \sqrt{1456.4043} \approx \$38.16$$

13. Kids.

$$\sigma^2 = Var(\text{Kids}) = (1-1.75)^2(0.5) + (2-1.75)^2(0.25) + (3-1.75)^2(0.25) = 0.6875$$

$$\sigma = SD(\text{Kids}) = \sqrt{Var(\text{Kids})} = \sqrt{0.6875} \approx 0.83 \text{ kids}$$

14. Darts.

$$\sigma^2 = Var(\text{Winnings}) = (95-17.20)^2(0.1) + (90-17.20)^2(0.09) + (85-17.20)^2(0.081) + (80-17.20)^2(0.0729) + (-20-17.20)^2(0.6561) \approx 2650.057$$

$$\sigma = SD(\text{Winnings}) = \sqrt{Var(\text{Winnings})} \approx \sqrt{2650.057} \approx \$51.48$$

15. Repairs.

a) $\mu = E(\text{Number of Repair Calls}) = 0(0.1) + 1(0.3) + 2(0.4) + 3(0.2) = 1.7$ calls

b)

$$\sigma^2 = Var(\text{Calls}) = (0-1.7)^2(0.1) + (1-1.7)^2(0.3) + (2-1.7)^2(0.4) + (3-1.7)^2(0.2) = 0.81$$

$$\sigma = SD(\text{Calls}) = \sqrt{Var(\text{Calls})} = \sqrt{0.81} = 0.9 \text{ calls}$$

16. Red lights.

a) $\mu = E(\text{Red lights}) = 0(0.05) + 1(0.25) + 2(0.35) + 3(0.15) + 4(0.15) + 5(0.05) = 2.25$ red lights

b)

$$\sigma^2 = Var(\text{Red lights}) = (0-2.25)^2(0.05) + (1-2.25)^2(0.25) + (2-2.25)^2(0.35) \\ + (3-2.25)^2(0.15) + (4-2.25)^2(0.15) + (5-2.25)^2(0.05) = 1.5875$$

$$\sigma = SD(\text{Red lights}) = \sqrt{Var(\text{Red lights})} = \sqrt{1.5875} \approx 1.26 \text{ red lights}$$

17. Defects.

The percentage of cars with *no* defects is 61%.

$$\mu = E(\text{Defects}) = 0(0.61) + 1(0.21) + 2(0.11) + 3(0.07) = 0.64 \text{ defects}$$

$$\sigma^2 = Var(\text{Defects}) = (0-0.64)^2(0.61) + (1-0.64)^2(0.21) \\ + (2-0.64)^2(0.11) + (3-0.64)^2(0.07) \approx 0.8704$$

$$\sigma = SD(\text{Defects}) = \sqrt{Var(\text{Defects})} \approx \sqrt{0.8704} \approx 0.93 \text{ defects}$$

18. Insurance.

a)

Profit	\$100	– \$9900	– \$2900
P(Profit)	0.9975	0.0005	0.002

b) $\mu = E(\text{Profit}) = 100(0.9975) - 9900(0.0005) - 2900(0.002) = \$89$

c)

$$\sigma^2 = Var(\text{Profit}) = (100-89)^2(0.9975) + (-9900-89)^2(0.0005) \\ + (-2900-89)^2(0.002) = 67{,}879$$

$$\sigma = SD(\text{Profit}) = \sqrt{Var(\text{Profit})} \approx \sqrt{67{,}879} \approx \$260.54$$

19. Cancelled flights.

a) $\mu = E(\text{gain}) = (-150)(0.20) + 100(0.80) = \$50$

b)

$$\sigma^2 = Var(\text{gain}) = (-150-50)^2(0.20) + (100-50)^2(0.80) = 10{,}000$$

$$\sigma = SD(\text{gain}) = \sqrt{Var(\text{gain})} \approx \sqrt{10{,}000} = \$100$$

20. Day trading again.

a) $\mu = E(\text{stock option}) = 1000(0.20) + 0(0.30) + 200(0.50) = \$300$

The trader should buy the stock option. Its expected value is \$300, and he only has to pay \$200 for it.

b) $\mu = E(\text{gain}) = 800(0.20) + (-200)(0.30) + 0(0.50) = \$100$

The trader expects to gain \$100. Notice that this is the same result as subtracting the \$200 price of the stock option from the \$300 expected value.

c)

$$\sigma^2 = Var(\text{gain}) = (800 - 100)^2(0.20) + (-200 - 100)^2(0.30) + (0 - 100)^2(0.50) = 130{,}000$$

$$\sigma = SD(\text{gain}) = \sqrt{Var(\text{gain})} \approx \sqrt{130{,}000} \approx \$360.56$$

Notice that the standard deviation of the trader's gain is the same as the standard deviation in value of the stock option.

21. Contest.

a) The two games are not independent. The probability that you win the second depends on whether or not you win the first.

b)

$$P(\text{losing both games}) = P(\text{losing the first})\, P(\text{losing the second} \mid \text{first was lost}) = (0.6)(0.7) = 0.42$$

c)

$$P(\text{winning both games}) = P(\text{winning the first})\, P(\text{winning the second} \mid \text{first was won}) = (0.4)(0.2) = 0.08$$

d)

X	0	1	2
$P(X = x)$	0.42	0.50	0.08

e)

$$\mu = E(X) = 0(0.42) + 1(0.50) + 2(0.08) = 0.66 \text{ games}$$

$$\sigma^2 = Var(X) = (0 - 0.66)^2(0.42) + (1 - 0.66)^2(0.50) + (2 - 0.66)^2(0.08) = 0.3844$$

$$\sigma = SD(X) = \sqrt{Var(X)} = \sqrt{0.3844} = 0.62 \text{ games}$$

22. Contracts.

a) The contracts are not independent. The probability that your company wins the second contract depends on whether or not your company wins the first contract.

b)

$$P(\text{getting both contracts}) = P(\text{getting \#1})\,P(\text{getting \#2} \mid \text{got \#1}) = (0.8)(0.2) = 0.16$$

c)

$$P(\text{getting no contract}) = P(\text{not getting \#1})\,P(\text{not getting \#2} \mid \text{didn't get \#1}) = (0.2)(0.7) = 0.14$$

d)

X	0	1	2
$P(X=x)$	0.14	0.70	0.16

e) $\mu = E(X) = 0(0.14) + 1(0.70) + 2(0.16) = 1.02$ contracts

$\sigma^2 = Var(X) = (0-1.02)^2(0.14) + (1-1.02)^2(0.70) + (2-1.02)^2(0.16) = 0.2996$

$\sigma = SD(X) = \sqrt{Var(X)} = \sqrt{0.2996} \approx 0.55$ contracts

23. Batteries.

a)

Number good	0	1	2
$P(\text{number good})$	$\left(\frac{3}{10}\right)\left(\frac{2}{9}\right) = \frac{6}{90}$	$\left(\frac{3}{10}\right)\left(\frac{7}{9}\right) + \left(\frac{7}{10}\right)\left(\frac{3}{9}\right) = \frac{42}{90}$	$\left(\frac{7}{10}\right)\left(\frac{6}{9}\right) = \frac{42}{90}$

b) $\mu = E(\text{number good}) = 0\left(\frac{6}{90}\right) + 1\left(\frac{42}{90}\right) + 2\left(\frac{42}{90}\right) = 1.4$ batteries

c)

$$\sigma^2 = Var(\text{number good}) = (0-1.4)^2\left(\frac{6}{90}\right) + (1-1.4)^2\left(\frac{42}{90}\right) + (2-1.4)^2\left(\frac{42}{90}\right) \approx 0.3733$$

$\sigma = SD(\text{number good}) = \sqrt{Var(\text{number good})} \approx \sqrt{0.3733} \approx 0.61$ batteries.

24. Kittens.

a)

Number of males	0	1	2
$P(\text{number of males})$	$\left(\frac{3}{7}\right)\left(\frac{2}{6}\right) = \frac{6}{42}$	$\left(\frac{4}{7}\right)\left(\frac{3}{6}\right) + \left(\frac{3}{7}\right)\left(\frac{4}{6}\right) = \frac{24}{42}$	$\left(\frac{4}{7}\right)\left(\frac{3}{6}\right) = \frac{12}{42}$

b) $\mu = E(\text{number of males}) = 0\left(\frac{6}{42}\right) + 1\left(\frac{24}{42}\right) + 2\left(\frac{12}{42}\right) \approx 1.14$ males

c) Answers may vary slightly (due to rounding of the mean)

$$\sigma^2 = Var(\text{number of males}) = (0-1.14)^2\left(\frac{6}{42}\right) + (1-1.14)^2\left(\frac{24}{42}\right) + (2-1.14)^2\left(\frac{12}{42}\right) \approx 0.4082$$

$$\sigma = SD(\text{number of males}) = \sqrt{Var(\text{number of males})} \approx \sqrt{0.4082} \approx 0.64 \text{ males}$$

25. Random variables.

a)

$$\mu = E(3X) = 3(E(X)) = 3(10) = 30$$
$$\sigma = SD(3X) = 3(SD(X)) = 3(2) = 6$$

b)

$$\mu = E(Y+6) = E(Y) + 6 = 20 + 6 = 26$$
$$\sigma = SD(Y+6) = SD(Y) = 5$$

c)

$$\mu = E(X+Y) = E(X) + E(Y) = 10 + 20 = 30$$
$$\sigma = SD(X+Y) = \sqrt{Var(X) + Var(Y)}$$
$$= \sqrt{2^2 + 5^2} \approx 5.39$$

d)

$$\mu = E(X-Y) = E(X) - E(Y) = 10 - 20 = -10$$
$$\sigma = SD(X-Y) = \sqrt{Var(X) + Var(Y)}$$
$$= \sqrt{2^2 + 5^2} \approx 5.39$$

e)

$$\mu = E(X_1 + X_2) = E(X) + E(X) = 10 + 10 = 20$$
$$\sigma = SD(X_1 + X_2) = \sqrt{Var(X) + Var(X)}$$
$$= \sqrt{2^2 + 2^2} \approx 2.83$$

26. Random variables.

a)

$$\mu = E(X-20) = E(X) - 20 = 80 - 20 = 60$$
$$\sigma = SD(X-20) = SD(X) = 12$$

b)

$$\mu = E(0.5Y) = 0.5(E(Y)) = 0.5(12) = 6$$
$$\sigma = SD(0.5Y) = 0.5(SD(Y)) = 0.5(3) = 1.5$$

c)

$$\mu = E(X+Y) = E(X) + E(Y) = 80 + 12 = 92$$
$$\sigma = SD(X+Y) = \sqrt{Var(X) + Var(Y)}$$
$$= \sqrt{12^2 + 3^2} \approx 12.37$$

d)

$$\mu = E(X-Y) = E(X) - E(Y) = 80 - 12 = 68$$
$$\sigma = SD(X-Y) = \sqrt{Var(X) + Var(Y)}$$
$$= \sqrt{12^2 + 3^2} \approx 12.37$$

e)

$$\mu = E(Y_1 + Y_2) = E(Y) + E(Y) = 12 + 12 = 24$$
$$\sigma = SD(Y_1 + Y_2) = \sqrt{Var(Y) + Var(Y)}$$
$$= \sqrt{3^2 + 3^2} \approx 4.24$$

27. Random variables.

a)

$$\mu = E(0.8Y) = 0.8(E(Y)) = 0.8(300) = 240$$
$$\sigma = SD(0.8Y) = 0.8(SD(Y)) = 0.8(16) = 12.8$$

b)

$$\mu = E(2X-100) = 2(E(X)) - 100 = 140$$
$$\sigma = SD(2X-100) = 2(SD(X)) = 2(12) = 24$$

c)

$$\mu = E(X+2Y) = E(X) + 2(E(Y))$$
$$= 120 + 2(300) = 720$$
$$\sigma = SD(X+2Y) = \sqrt{Var(X) + 2^2 Var(Y)}$$
$$= \sqrt{12^2 + 2^2(16^2)} \approx 34.18$$

d)

$$\mu = E(3X-Y) = 3(E(X)) - E(Y)$$
$$= 3(120) - 300 = 60$$
$$\sigma = SD(3X-Y) = \sqrt{3^2 Var(X) + Var(Y)}$$
$$= \sqrt{3^2(12^2) + 16^2} \approx 39.40$$

e)

$$\mu = E(Y_1 + Y_2) = E(Y) + E(Y) = 300 + 300 = 600$$
$$\sigma = SD(Y_1 + Y_2) = \sqrt{Var(Y) + Var(Y)}$$
$$= \sqrt{16^2 + 16^2} \approx 22.63$$

28. Random variables.

a)

$$\mu = E(2Y+20) = 2(E(Y)) + 20$$
$$= 2(12) + 20 = 44$$
$$\sigma = SD(2Y+20) = 2(SD(Y)) = 2(3) = 6$$

b)

$$\mu = E(3X) = 3(E(X)) = 3(80) = 240$$
$$\sigma = SD(3X) = 3(SD(X)) = 3(12) = 36$$

c)

$$\mu = E(0.25X + Y) = 0.25(E(X)) + E(Y)$$
$$= 0.25(80) + 12 = 32$$
$$\sigma = SD(0.25X + Y) = \sqrt{0.25^2 Var(X) + Var(Y)}$$
$$= \sqrt{0.25^2(12^2) + 3^2} \approx 4.24$$

d)

$$\mu = E(X - 5Y) = E(X) - 5(E(Y))$$
$$= 80 - 5(12) = 20$$
$$\sigma = SD(X - 5Y) = \sqrt{Var(X) + 5^2 Var(Y)}$$
$$= \sqrt{12^2 + 5^2(3^2)} \approx 19.21$$

e)

$$\mu = E(X_1 + X_2 + X_3) = E(X) + E(X) + E(X) = 80 + 80 + 80 = 240$$
$$\sigma = SD(X_1 + X_2 + X_3) = \sqrt{Var(X_1) + Var(X_2) + Var(X_3)}$$
$$= \sqrt{12^2 + 12^2 + 12^2} \approx 20.78$$

29. Eggs.

a) $\mu = E(\text{Broken eggs in 3 dozen}) = 3(E(\text{Broken eggs in 1 dozen})) = 3(0.6) = 1.8$ eggs

b) $\sigma = SD(\text{Broken eggs in 3 dozen}) = \sqrt{0.5^2 + 0.5^2 + 0.5^2} \approx 0.87$ eggs

c) The cartons of eggs must be independent of each other.

30. Garden.

a) $\mu = E(\text{bad seeds in 5 packets}) = 5(E(\text{bad seeds in 1 packet})) = 5(2) = 10$ bad seeds

b) $\sigma = SD(\text{bad seeds in 5 packets}) = \sqrt{1.2^2 + 1.2^2 + 1.2^2 + 1.2^2 + 1.2^2} \approx 2.68$ bad seeds

c) The packets of seeds must be independent of each other. If you buy an assortment of seeds, this assumption is probably OK. If you buy all of one type of seed, the store probably has seed packets from the same batch or lot. If some are bad, the others might tend to be bad as well.

31. Repair calls.

$$\mu = E(\text{calls in 8 hours}) = 8(E(\text{calls in 1 hour}) = 8(1.7) = 13.6 \text{ calls}$$

$$\sigma = SD(\text{calls in 8 hours}) = \sqrt{8(Var(\text{calls in 1 hour}))} = \sqrt{8(0.9)^2} \approx 2.55 \text{ calls}$$

This is only valid if the hours are independent of one another.

32. Stop!

$$\mu = E(\text{red lights in 5 days}) = 5(E(\text{red lights each day})) = 5(2.25) = 11.25 \text{ red lights}$$

$$\sigma = SD(\text{red lights in 5 days}) = \sqrt{5(Var(\text{red lights each day})} = \sqrt{5(1.26)^2} \approx 2.82 \text{ red lights}$$

Standard deviation may vary slightly due to rounding of the standard deviation of the number of red lights each day, and may only be calculated if the days are independent of each other. This seems reasonable.

33. Tickets.

a)

$$\mu = E(\text{tickets for 18 trucks}) = 18(E(\text{tickets for one truck})) = 18(1.3) = 23.4 \text{ tickets}$$

$$\sigma = SD(\text{tickets for 18 trucks}) = \sqrt{18(Var(\text{tickets for one truck})} = \sqrt{18(0.7)^2} \approx 2.97 \text{ tickets}$$

b) We are assuming that trucks are ticketed independently.

34. Donations.

a)

$$\mu = E(\text{pledges from 120 people}) = 120(E(\text{pledge from one person})) = 120(32) = \$3840$$

$$\sigma = SD(\text{pledges from 120 people}) = \sqrt{120(Var(\text{pledge from one person})} = \sqrt{120(54)^2} \approx \$591.54$$

b) We are assuming that callers make pledges independently from one another.

35. Fire!

a) The standard deviation is large because the profits on insurance are highly variable. Although there will be many small gains, there will occasionally be large losses, when the insurance company has to pay a claim.

b)

$$\mu = E(\text{two policies}) = 2(E(\text{one policy})) = 2(150) = \$300$$

$$\sigma = SD(\text{two policies}) = \sqrt{2(Var(\text{one policy}))} = \sqrt{2(6000^2)} \approx \$8{,}485.28$$

c)

$$\mu = E(\text{10,000 policies}) = 10{,}000(E(\text{one policy})) = 10{,}000(150) = \$1{,}500{,}000$$

$$\sigma = SD(\text{10,000 policies}) = \sqrt{10{,}000(Var(\text{one policy}))} = \sqrt{10{,}000(6000^2)} = \$600{,}000$$

d) If the company sells 10,000 policies, they are likely to be successful. A profit of \$0, is 2.5 standard deviations below the expected profit. This is unlikely to happen.
However, if the company sells fewer policies, then the likelihood of turning a profit decreases. In an extreme case, where only two policies are sold, a profit of \$0 is more likely, being only a small fraction of a standard deviation below the mean.

e) This analysis depends on each of the policies being independent from each other. This assumption of independence may be violated if there are many fire insurance claims as a result of a forest fire, or other natural disaster.

36. Casino.

a) The standard deviation of the slot machine payouts is large because most players will lose their dollar, but a few large payouts are expected. The payouts are highly variable.

b)

$$\mu = E(\text{profit from 5 plays}) = 5(E(\text{profit from one play})) = 5(0.08) = \$0.40$$

$$\sigma = SD(\text{profit from 5 plays}) = \sqrt{5(Var(\text{profit from one play}))} = \sqrt{5(120^2)} \approx \$268.33$$

c)

$$\mu = E(\text{profit from 1000 plays}) = 1000(E(\text{profit from one play})) = 1000(0.08) = \$80$$

$$\sigma = SD(\text{profit from 1000 plays}) = \sqrt{1000(Var(\text{profit from one play}))} = \sqrt{1000(120^2)} \approx \$3{,}794.73$$

d) If the machine is played only 1000 times a day, the chance of being profitable isn't as high as the casino might like, since \$80 is only approximately 0.02 standard deviations above 0. But if the casino has many slot machines, the chances of being profitable will go up.

37. Cereal.

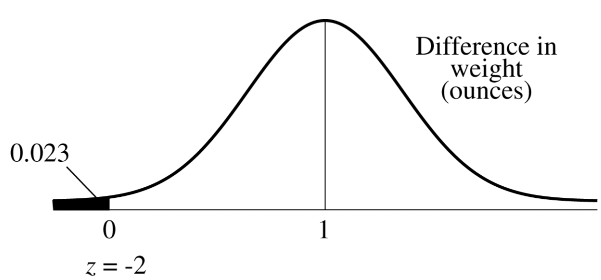
a) $E(\text{large bowl} - \text{small bowl}) = E(\text{large bowl}) - E(\text{small bowl}) = 2.5 - 1.5 = 1$ ounce

b) $\sigma = SD(\text{large bowl} - \text{small bowl}) = \sqrt{Var(\text{large}) + Var(\text{small})} = \sqrt{0.4^2 + 0.3^2} = 0.5$ ounces

c)

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{0 - 1}{0.5} = -2$$



The small bowl will contain more cereal than the large bowl when the difference between the amounts is less than 0. According to the Normal model, the probability of this occurring is approximately 0.023.

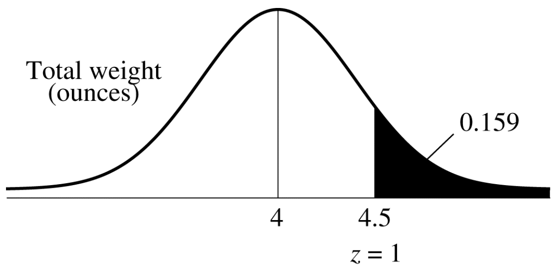
d)

$$\mu = E(\text{large bowl} + \text{small bowl}) = E(\text{large bowl}) + E(\text{small bowl}) = 2.5 + 1.5 = 4 \text{ ounce}$$

$$\sigma = SD(\text{large bowl} + \text{small bowl}) = \sqrt{Var(\text{large}) + Var(\text{small})} = \sqrt{0.4^2 + 0.3^2} = 0.5 \text{ ounces}$$

e)

$$z = \frac{x-\mu}{\sigma}$$
$$z = \frac{4.5-4}{0.5} = 1$$



According to the Normal model, the probability that the total weight of cereal in the two bowls is more than 4.5 ounces is approximately 0.159.

f)

$$\mu = E(\text{box} - \text{large} - \text{small}) = E(\text{box}) - E(\text{large}) - E(\text{small}) = 16.3 - 2.5 - 1.5 = 12.3 \text{ ounces}$$
$$\sigma = SD(\text{box} - \text{large} - \text{small}) = \sqrt{Var(\text{box}) + Var(\text{large}) + Var(\text{small})}$$
$$= \sqrt{0.2^2 + 0.3^2 + 0.4^2} \approx 0.54 \text{ ounces}$$

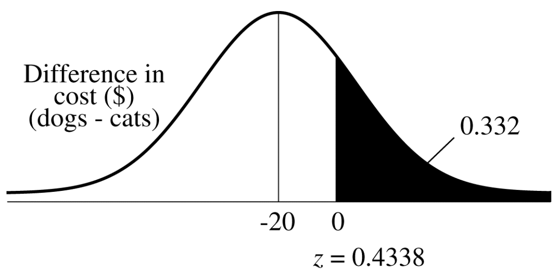
38. Pets.

a) $\mu = E(\text{dogs} - \text{cats}) = E(\text{dogs}) - E(\text{cats}) = 100 - 120 = -\$20$

b) $\sigma = SD(\text{dogs} - \text{cats}) = \sqrt{Var(\text{dogs}) + Var(\text{cats})} = \sqrt{30^2 + 35^2} \approx \$46.10$

c)

$$z = \frac{x-\mu}{\sigma}$$
$$z = \frac{0-(-20)}{46.10}$$
$$z = 0.4338$$



The expected cost of the dog is greater than that of the cat when the difference in cost is positive (greater than 0). According to the Normal model, the probability of this occurring is about 0.332.

39. More cereal.

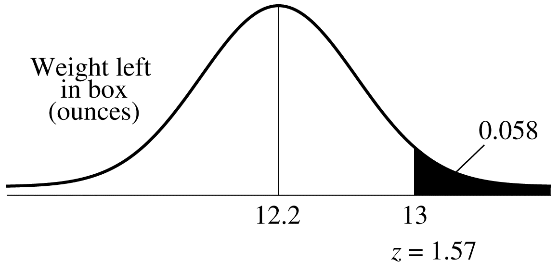
a) $\mu = E(\text{box} - \text{large} - \text{small}) = E(\text{box}) - E(\text{large}) - E(\text{small}) = 16.2 - 2.5 - 1.5 = 12.2 \text{ ounces}$

b)

$$\sigma = SD(\text{box} - \text{large} - \text{small}) = \sqrt{Var(\text{box}) + Var(\text{large}) + Var(\text{small})}$$
$$= \sqrt{0.1^2 + 0.3^2 + 0.4^2} \approx 0.51 \text{ ounces}$$

c)

$$z = \frac{x-\mu}{\sigma}$$
$$z = \frac{13-12.2}{0.51}$$
$$z = 1.57$$



According to the Normal model, the probability that the box contains more than 13 ounces is about 0.058.

40. More pets.

a) Let X = cost for a dog, and let Y = cost for a cat.

Total cost = $X + X + Y$

b)

$$\mu = E(X + X + Y) = E(X) + E(X) + E(Y) = 100 + 100 + 120 = \$320$$

$$\sigma = SD(X + X + Y) = \sqrt{Var(X) + Var(X) + Var(Y)}$$

$$= \sqrt{30^2 + 30^2 + 35^2} = \$55$$

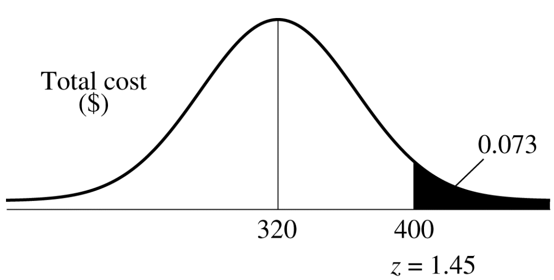
Since the models for individual pets are Normal, the model for total costs is Normal with mean \$320 and standard deviation \$55.

c)

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{400 - 320}{55}$$

$$z = 1.45$$



According to the Normal model, the probability that the total cost of two dogs and a cat is more than \$400 is approximately 0.073.

41. Medley.

a)

$$\mu = E(\#1+\#2+\#3+\#4) = E(\#1) + E(\#2) + E(\#3) + E(\#4)$$

$$= 50.72 + 55.51 + 49.43 + 44.91 = 200.57 \text{ seconds}$$

$$\sigma = SD(\#1+\#2+\#3+\#4) = \sqrt{Var(\#1) + Var(\#2) + Var(\#3) + Var(\#4)}$$

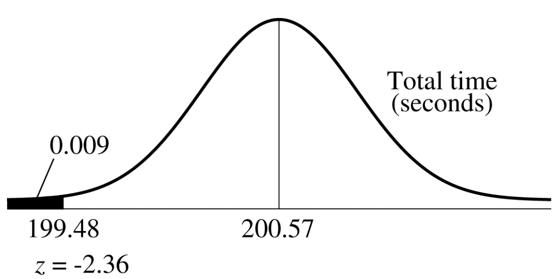
$$= \sqrt{0.24^2 + 0.22^2 + 0.25^2 + 0.21^2} \approx 0.46 \text{ seconds}$$

b)

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{199.48 - 200.57}{0.461}$$

$$z = -2.36$$



The team is not likely to swim faster than their best time. According to the Normal model, they are only expected to swim that fast or faster about 0.9% of the time.

42. Bikes.

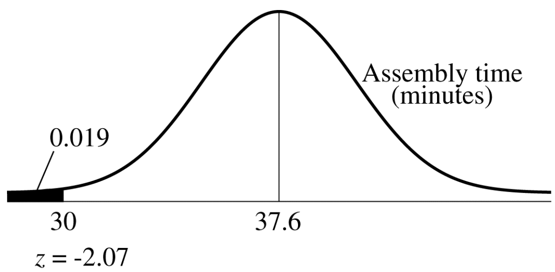
a)

$$\mu = E(\text{unpack} + \text{assembly} + \text{tuning}) = E(\text{unpack}) + E(\text{assembly}) + E(\text{tuning})$$
$$= 3.5 + 21.8 + 12.3 = 37.6 \text{ minutes}$$

$$\sigma = SD(\text{unpack} + \text{assembly} + \text{tuning}) = \sqrt{Var(\text{unpack}) + Var(\text{assembly}) + Var(\text{tuning})}$$
$$= \sqrt{0.7^2 + 2.4^2 + 2.7^2} \approx 3.7 \text{ minutes}$$

b)

$$z = \frac{x-\mu}{\sigma}$$
$$z = \frac{30-37.6}{3.68}$$
$$z = -2.07$$



The bike is not likely to be ready on time. According to the Normal model, the probability that an assembly is completed in under 30 minutes is about 0.019.

43. Farmer's market.

a) Let $A =$ price of a pound of apples, and let $P =$ price of a pound of potatoes.

Profit $= 100A + 50P - 2$

b) $\mu = E(100A + 50P - 2) = 100(E(A)) + 50(E(P)) - 2 = 100(0.5) + 50(0.3) - 2 = \$63$

c) $\sigma = SD(100A + 50P - 2) = \sqrt{100^2(Var(A)) + 50^2(Var(P))} = \sqrt{100^2(0.2^2) + 50^2(0.1^2)} \approx \$20.62$

d) No assumptions are necessary to compute the mean. To compute the standard deviation, independent market prices must be assumed.

44. Bike sale.

a) Let $B =$ number of basic bikes sold, and let $D =$number of deluxe bikes sold.

Net Profit $= 120B + 150D - 200$

b) $\mu = E(120B + 150D - 200) = 120(E(B)) + 150(E(D)) - 200 = 120(5.4) + 150(3.2) - 200 = \$928$

c)

$$\sigma = SD(120B + 150D - 200) = \sqrt{120^2(Var(B)) + 150^2(Var(D))}$$
$$= \sqrt{120^2(1.2^2) + 150^2(0.8^2)} \approx \$187.45$$

d) No assumptions are necessary to compute the mean. To compute the standard deviation, independent sales must be assumed.

45. Coffee and doughnuts.

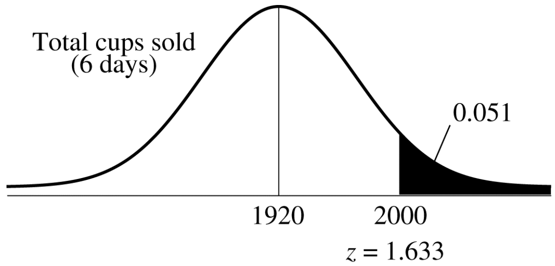
a)

$$\mu = E(\text{cups sold in 6 days}) = 6(E(\text{cups sold in 1 day})) = 6(320) = 1920 \text{ cups}$$

$$\sigma = SD(\text{cups sold in 6 days}) = \sqrt{6(Var(\text{cups sold in 1 day})} = \sqrt{6(20)^2} \approx 48.99 \text{ cups}$$

The distribution of total coffee sales for 6 days has distribution $N(1920{,}48.99)$.

$$z = \frac{x - \mu}{\sigma}$$
$$z = \frac{2000 - 1920}{48.99}$$
$$z = 1.633$$



According to the Normal model, the probability that he will sell more than 2000 cups of coffee in a week is approximately 0.051.

b) Let C = the number of cups of coffee sold. Let D = the number of doughnuts sold.

$$\mu = E(50C + 40D) = 0.50(E(C)) + 0.40(E(D)) = 0.50(320) + 0.40(150) = \$220$$

$$\sigma = SD(0.50C + 0.40D) = \sqrt{0.50^2(Var(C)) + 0.40^2(Var(D))} = \sqrt{0.50^2(20^2) + 0.40^2(12^2)} \approx \$11.09$$

The day's profit can be modeled by $N(220,11.09)$. A day's profit of \$300 is over 7 standard deviations above the mean. This is extremely unlikely. It would not be reasonable for the shop owner to expect the day's profit to exceed \$300.

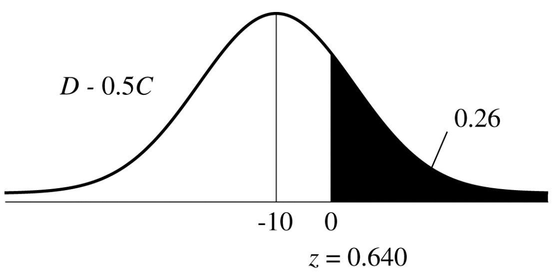
c) Consider the difference $D - 0.5C$. When this difference is greater than zero, the number of doughnuts sold is greater than half the number of cups of coffee sold.

$$\mu = E(D - 0.5C) = (E(D)) - 0.5(E(C)) = 150 + 0.5(320) = -\$10$$

$$\sigma = SD(D - 0.5C) = \sqrt{(Var(D)) + 0.5(Var(C))} = \sqrt{(12^2) + 0.5^2(20^2)} \approx \$15.62$$

The difference $D - 0.5C$ can be modeled by $N(-10, 15.62)$.

$$z = \frac{x - \mu}{\sigma}$$
$$z = \frac{0 - (10)}{15.62}$$
$$z = 0.640$$



According to the Normal model, the probability that the shop owner will sell a doughnut to more than half of the coffee customers is approximately 0.26.

46. Weightlifting.

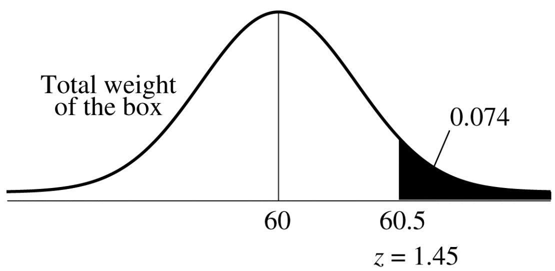
a) Let T = the true weight of a 20-pound weight.
Let F = the true weight of a 5-pound weight.
Let B = the true weight of the bar.

$$\mu = E(\text{Total weight}) = 2E(T) + 4E(F) = 2(20) + 4(5) = 60 \text{ pounds}$$

$$\sigma = SD(\text{Total}) = \sqrt{2(Var(T)) + 4(Var(F))} = \sqrt{2(0.2^2) + 4(0.1^2)} = \sqrt{0.12} \approx 0.346 \text{ pounds}$$

Assuming that the true weights of each piece are independent of one another, the total weight of the box can be modeled by $N(60,0.346)$.

$$z = \frac{x-\mu}{\sigma}$$
$$z = \frac{60-60.5}{0.346}$$
$$z = 1.45$$



According to the Normal model, the probability that the total weight in the shipping box exceeds 60.5 pounds is approximately 0.074.

b) $\text{Cost} = 0.40(T+T+F+F+F+F)+0.50(B)+6$

$$\mu = E(\text{Cost}) = 0.40(E(T+T+F+F+F+F))+0.50(E(B))+6$$
$$= 0.40(60)+0.50(10)+6 = \$35$$

$$\sigma = SD(\text{Cost}) = \sqrt{0.40^2(Var(\text{Total weight of box}))+0.50^2(Var(B))}$$
$$= \sqrt{0.40^2(0.12)+0.50^2(0.25^2)} \approx \$0.187$$

The shipping cost for the starter set has mean \$35 and standard deviation \$0.187.

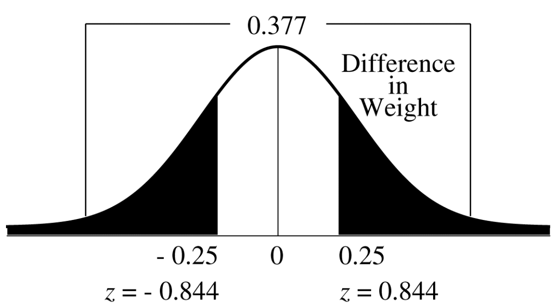
c) Consider the expression $T-(F+F+F+F)$, which represents the difference in weight between a 20-pound weight and four 5-pound weights. We are interested in the distribution of this difference, so that we can find the probability that the difference is greater than 0.25 pounds.

$$\mu = E(T-(F+F+F+F)) = E(T)-4E(F) = 20-4(5) = 0 \text{ pounds}$$

$$\sigma = SD(T-(F+F+F+F)) = \sqrt{(Var(T))+4(Var(F))} = \sqrt{(0.2^2)+4(0.1^2)} \approx 0.2828 \text{ pounds}.$$

The difference in weight can be modeled by $N(0, 0.2828)$.

$$z = \frac{x-\mu}{\sigma}$$
$$z = \frac{0-0.25}{0.2828}$$
$$z = 0.884$$



According to the Normal model, the probability that the difference in weight between one 20-pound weight and four 5-pound weights is greater than 0.25 pounds is 0.377.